Marginal Utility and New Saint Petersburg Paradoxes

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Adam Smith may have read Daniel Bernoulli's 1738 essay on risk, and Smith modified his view on risk while teaching jurisprudence to two Russian students, this essay argues. The matter is important because William Stanley Jevons read Adam Smith closely, of course, but Jevons did not read Daniel Bernoulli, and Jevons convinced Alfred Marshall that the concept of marginal utility did not need the advanced mathematical probability which they could have found in Bernoulli. Jevons thought arguments in English prose, like Smith's arguments, together with the very simple mathematics of Gregory King were sufficient for discussion of moral expectation (what was later called marginal utility). We begin with the tantalizing suggestion that Adam Smith modeled his famous notion of the invisible hand upon Daniel Bernoulli's also famous essay about risk. The resemblance between the two was striking. Bernoulli's essay gained the nickname "The Saint Petersburg Paradox" because the author was at the Imperial Academy of Sciences in Saint Petersburg when he authored the essay and because he published his essay with the proceedings of that same Imperial Academy in Saint Petersburg. Evidence for the connection between Bernoulli and Smith is only circumstantial, however. This present essay lays out the circumstantial evidence, comparing passages from Bernoulli on risk and from Smith on the invisible hand and on what Smith called the lottery of employment.

Keywords: D. Bernoulli, W. Jevons, invisible hand, marginal utility, risk, A. Smith.

Предельная полезность и новые Санкт-Петербургские парадоксы

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Как утверждается в данном исследовании, Адам Смит мог быть знаком с эссе Даниила Бернулли о риске, опубликованном в 1738 г., и изменить свои взгляды на риск во время преподавания юриспруденции двум русским студентам. Этот вопрос важен потому, что У. С. Джевонс, несомненно, внимательно читал Адама Смита, но не читал Даниила Бернулли. Джевонс убедил Альфреда Маршалла, что концепция предельной полезности не нуждается в математическом анализе или развитой математике вероятности, которые они могли бы найти у Бернулли. Джевонс считал, что аргументы в английской литературе, подобные аргументам Смита, вместе с очень простой математикой Грегори

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Кинга достаточны для обсуждения морального ожидания (того, что позже было названо предельной полезностью). Высказывается предположение, что Адам Смит смоделировал свое знаменитое понятие «невидимой руки» на основе также знаменитого сочинения Даниила Бернулли о риске. Сходство между ними было поразительным. Среди специалистов высказанные в эссе Бернулли идеи получили название «Санкт-Петербургский парадокс», потому что автор находился в Императорской Академии наук в Санкт-Петербурге, когда писал эссе, и потому что он опубликовал свое эссе в сборнике трудов той же Императорской Академии в Санкт-Петербурге. Однако доказательства связи между Бернулли и Смитом носят лишь косвенный характер. В настоящем исследовании мы приводим эти косвенные доказательства, сравнивая отрывки из сочинения Бернулли о риске, а также из труда Смита о «невидимой руке» и о том, что Смит называл трудовой лотереей.

Ключевые слова: Д.Бернулли, У.Джевонс, невидимая рука рынка, предельная полезность, риск, А. Смит.

The 18th-century Scottish moral philosopher Adam Smith may have read a seminal 1738 essay which the Swiss mathematician Daniel Bernoulli published about risk. If Smith read Bernoulli, however, and if Smith echoed Bernoulli, then that went unnoticed by William Stanley Jevons and Alfred Marshall. The question is significant because answering it will lead us to think differently about Jevons, Marshall, and marginal utility.

Let us start with Jevons. He is still widely credited in the English-speaking world with inventing what came to be called the concept of marginal utility. Here for instance is the current definition of the word "marginalism" in The Oxford English Dictionary. "Marginalism was developed independently by W.S. Jevons (1835–1882), C. Menger (1840–1921), and M. E. L. Walras (1834–1910). It broke with the classical definition of value as the quantity of labour embodied in a product, and was thus an important component in the development of neoclassical economics." The dictionary's definition of marginalism certainly reflected — and it may have been derived from — the work of the great 19th-century English economist Alfred Marshall. He and many other economists thereafter said that Jevons was a "marginalist," and that Jevons was part of "the marginalist revolution." Here was Marshall's definition of marginal utility. Speaking of an imaginary customer, Marshall wrote as follows. "That part of the thing which he is only just induced to purchase may be called his marginal purchase, because he is on the margin of doubt whether it is worth his while to incur the outlay required to obtain it. And the utility of his marginal purchase may be called the marginal utility of the thing to him." Marshall often put his main points into footnotes, and Marshall wrote as follows in a footnote here. "The marginal utility of a thing to anyone diminishes with every increase in the amount of it he already has"¹.

According to Jevons, value was a subjective thing, not objective. Marshall agreed. He said later that decisions at the margin often determined value. The circumstances of the buyer determined the buyer's decision — what he or she was willing to pay for an additional or marginal item — and those circumstances were individual and idiosyncratic. The individual buyer based such decisions in large part on subjective feelings². As we will see now, Bernoulli's earlier concept of moral expectation was similar to marginal utility.

¹ Marshall A. Principles of Economics. London, 1890. P.93.

² Peart S. J., Levy D. M. Menger and Jevons: beliefs and things // The Review of Austrian Economics. 2023. Vol. 36, issue 2. P.271–287.

Daniel Bernoulli, the Saint Petersburg Paradox, and Marginal Utility

The fact is that Daniel Bernoulli invented the concept of marginal utility before Jevons. Bernoulli published it in 1738 in his essay "Exposition of a New Theory on the Measurement of Risk". His definition was full and clear although he used the phrase "moral expectation," and he did not use the phrase "marginal utility." Jevons skipped over Bernoulli's essay on risk, however. Jevons did not read it. Why?

Bernoulli wrote his 1738 essay in Latin. He posed what came to be called the "Saint Petersburg Paradox". The paradox involved gambling on a coin toss. A coin toss posed a specific example of risk when it involved a bet on the outcome of the toss. Value in the Saint Petersburg Paradox differed from person to person because it reflected the differing estimates of bettors concerning the effect of losing the bet or winning it. For instance, a rich bettor valued both the cost of the bet and the payoff less than a poor bettor would, and a poor bettor valued both of them more than the rich bettor. Value was therefore a subjective judgment which depended on the circumstance of the individual, said Bernoulli. However, Bernoulli's essay was at first construed only as a solution to a difficult puzzle in mathematics. Bernoulli was born into a family of mathematical geniuses, and he came to the Saint Petersburg Paradox from a study of mathematical probability.

A. A. Kundryavtsev said that Bernoulli's argument began as mathematics and only later became economics. The present essay takes Kundryavtsev's argument further. We show here that the Bernoulli argument in the Saint Petersburg Paradox was connected with the specific concept of marginal utility, a crucial concept in 19th-century economics³. The Bernoulli paragraph which related to marginal utility ran as follows.

\$3. To make this clear it is perhaps advisable to consider the following example: Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats. If I am not wrong then it seems clear that all men cannot use the same rule to evaluate the gamble. The rule established in § 1 must, therefore, be discarded. But anyone who considers the problem with perspicacity and interest will ascertain that the concept of value which we have used in this rule may be defined in a way which renders the entire procedure universally acceptable without reservation. To do this the determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus, there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount⁴.

Academic essays tend to emphasize reason. Bernoulli's essay emphasized subjectivity, however. Value in the Saint Petersburg Paradox, he said, depended on subjective emotion. In the Saint Petersburg Paradox, at least in Saint Petersburg itself, bettors tossed a supposedly fair gold or silver coin until the first time it came up eagles. Russian coins usually

³ *Kudryavtsev A.A.* St. Petersburg Paradox and its Significance for Economic Theory // Vestnik of Saint Petersburg State University. Series 5. Economics. 2013. Issue 3. P.41–55.

⁴ *Bernoulli D*. Exposition of a New Theory on the Measurement of Risk // Econometrica. 1954. Vol. 22, issue 1. P.23–24.

had the doubled-headed Byzantine eagle on one side and an inscription on the other side. They usually did not have heads inscribed on them. Some coins did, but most did not, and as that was true before so it is true now. "Fair" meant a coin was equally likely to land on either of the two sides when tossed.

Mints made coins by hand in past time. The word "coin" (or "quoin") meant "wedge" in French because the mold with which they stamped out a coin was in the form of a wedge. These real handmade coins were beautiful and interesting but they were seldom precisely fair. Each one was unique, each one a bit different from all the others. They were thus almost all biased to some degree. That increased the element of risk for bettors.

The payoff of Bernoulli's imagined bet increased exponentially. If the coin came up eagles on the first toss, then the payoff was two such coins; if the first time it came up eagles was on the second toss, the payoff was four such coins; and if it was on the n^{th} toss, then the payoff was two times n in such coins.

By taking into account the subjective feelings of the bettors, Daniel Bernoulli evaluated the risk for gamblers in this game. The winning of a gold or silver coin was of much more value to a poor bettor than to a rich one. The poor bettor additionally could less easily afford the cost of the bet than a rich bettor. The distinguished 20th-century Yale statistician Leonard J. Savage said Bernoulli's concept of moral expectation was "mystical"⁵. We will turn to an evaluation of these subjective feelings in the Saint Petersburg Paradox presently. Let us first return to Adam Smith.

Did Adam Smith read Bernoulli's 1738 essay on risk?

To seek an answer to that question, let us first see whether Smith had an opportunity to read Bernoulli's essay. We find a clear answer to that second question in the Archives of Glasgow University Library. Remember that Adam Smith was a professor of moral philosophy at Glasgow University, and later — lord rector of that same university. The Archives there contain conclusive evidence that Smith had an opportunity to read Bernoulli's 1738 essay on risk. The proceedings of the Saint Petersburg Academy of Sciences published the essay, and the library of the University of Glasgow had a copy of that volume of the proceedings which Smith therefore could have read while he was writing his book on moral sentiments. Of course, this evidence is not conclusive proof that Smith read Bernoulli in the university library. Smith could instead have read the essay elsewhere. Also, and on the other hand, his having the opportunity to read the essay does not prove that he did read it. However, that information from the university Archives made his reading of Bernoulli a definite possibility. This and the subsequent materials cited here from the Archive of the University of Glasgow were supplied by Robert MacLean, Assistant Librarian (rare books & engagement). He wrote a personal email of October 12, 2023, doing so in response to a query from the author of this present essay who here expresses his gratitude for this expert's help.

Remember next the famous passage from the first chapter of the fourth book in Adam Smith's *Theory of Moral Sentiments*, first published in 1759. To refresh your memory, here is the passage. Maybe it is the most famous passage in the whole literature of political economy. Smith imagined a rich, rapacious landowner.

⁵ Savage L. J. The Foundations of Statistics. New York, 1972. P.93.

"The capacity of his stomach bears no proportion to the immensity of his desires, and will receive no more than that of the meanest peasant. The rest he is obliged to distribute among those, who prepare, in the nicest manner, that little which he himself makes use of, among those who fit up the palace in which this little is to be consumed, among those who provide and keep in order all the different baubles and trinkets, which are employed in the economy of greatness; all of whom thus derive from his luxury and caprice, that share of the necessaries of life, which they would in vain have expected from his humanity or his justice. The produce of the soil maintains at all times nearly that number of inhabitants which it is capable of maintaining. The rich only select from the heap what is most precious and agreeable. They consume little more than the poor, and in spite of their natural selfishness and rapacity, though they mean only their own conveniency, though the sole end which they propose from the labours of all the thousands whom they employ, be the gratification of their own vain and insatiable desires, they divide with the poor the produce of all their improvements. They are led by an invisible hand to make nearly the same distribution of the necessaries of life, which would have been made, had the earth been divided into equal portions among all its inhabitants, and thus without intending it, without knowing it, advance the interest of the society, and afford means to the multiplication of the species"6.

Compare Smith's argument in English with Bernoulli's earlier reasoning as represented in English (Smith would have read it in Latin). On the one hand, Smith's argument was very similar to that of Bernoulli. A rich man and a poor man view value differently because of their differing circumstances. Smith's very wording was similar to Bernoulli's. That was the matter in a nutshell. That similarity in topic and wording, and on the other hand, the information from the Archive of the University of Glasgow, are the two principal circumstances which argue for Smith's having read Bernoulli. There is still more circumstantial evidence, however.

Two Russian Students Studied with Adam Smith in Glasgow University

Here is another piece of circumstantial evidence for Smith's having read Daniel Bernoulli on risk. This, however, relates more to *The Wealth of Nations* than to *The Theory of Moral Sentiments*. Nearly a century ago, Michael P. Alekseev, a Soviet scholar, published a brilliant essay "Adam Smith and His Russian Admirers of the Eighteenth Century". He related facts as follows. In 1761, at the instigation of the curator of Moscow University, the Russian government, which was then seated in Saint Petersburg, allowed two young Russian men to study at the University of Glasgow in Scotland. They were Semen Efimovich Desnitskii (1740–1789) and Ivan Andreevich Tret'iakov (1735–1776). They studied law and philosophy at Glasgow with Adam Smith, who was then the author of *The Theory of Moral Sentiments*, published in 1759, and not yet of *The Wealth of Nations*, published in 1776. Alekseev established these facts. No one disputes them⁷.

After Desnitskii and Tret'iakov studied with Smith, they duly returned to Saint Petersburg with their Scottish diplomas, and they then embellished the Russian reputation of Adam Smith. Desnitskii was especially active as a Russian disciple of Smith. Here are comments about him from a very recent scholarly article.

⁶ Smith A. The Theory of Moral Sentiments. London, 1761. P.273–274.

⁷ Alekseev M.P. Adam Smith and his Russian admirers of the eighteenth century // Econ Journal Watch Scholarly Comments on Academic Economics. 2018. Vol. 15, issue 3. P. 354–364.

Desnitskii was considered "the father of Russian jurisprudence". During his career as a professor of law at Moscow University, Desnitskii was the first Russian who taught Roman law using the Russian language. He thus followed the "New Lights" who began lecturing in the English language at Glasgow University. He completed his legal studies at Glasgow University where he was awarded the title of Doctor of Laws"⁸.

The Archive of the University of Glasgow also contains information about the studies of these two Russian students. The Archive has, for instance, the joint petition from them in which they requested that they be awarded their degrees in law⁹.

Students Aleksei Kuznetsov and Iana Negina, visiting the Archive of the Academy of Sciences in Saint Petersburg, recently searched there for materials pursuant to this present essay. I am grateful for their help. Their search did not reveal any documents which shed light on Smith's possibly having read Bernoulli on risk. However, the Archive in Saint Petersburg did have documents about the two Russian students. The two young men ran out of money, the documents say, and that was why they had to return home quickly. Remember that Adam Smith and Daniel Bernoulli both had lasting impacts in Russia. The famous poet Alexander Pushkin cited Adam Smith in the great poem *Eugene Onegin*, of course. Pushkin and other Russians could read Smith's *Theory of Moral Sentiments* in its French translation. Russians could also learn about Smith from Henri Storch¹⁰ who wrote a summary of economic thought, which was popular in Russia. Bernoulli was in contact with many important people in Russia, and the Academy of Sciences sent books and papers to the Russian ambassador in London although it is not clear which books or papers were sent.

Let us summarize. When writing *The Theory of Moral Sentiments*, Smith had an opportunity to read Bernoulli on risk. So did his students. Let us now go forward.

Smith also had a motive to read Bernoulli on risk. Smith was certainly interested in value and in gambling where risk rendered value uncertain. We can see that in his writing about gambling in Chapter I of Part VII of *The Theory of Moral Sentiments*. Smith recommended methods to allay the mental anguish associated with mischance and loss. That passage could be a commentary on Bernoulli. Alas, Smith here discussed Epictetus and the Stoics instead. Smith recommended methods to allay the mental anguish associated with mischance and loss.

Smith deepened and changed his understanding of risk later as, for instance, when he wrote *The Wealth of Nations*, published in 1776. In *The Wealth of Nations*, he made a new remark about an invisible hand. The further remark showed his readers how much Smith had changed his views over time. Here is that further invisible hand remark in *The Wealth of Nations*.

"But the annual revenue of every society is always precisely equal to the exchangeable value of the whole annual produce of its industry, or rather is precisely the same thing with that

⁸ *Marchevský O., Zákutná S.* The ideas of the Scottish Enlightenment in Russia: Adam Smith and Semyon Efimovich Desnitsky on the Philosophy of History. Studies in East European Thought. Available at: https://doi.org/10.1007/s11212-023-09582-6 (accessed: 14.10.2023).

⁹ [Letter from Simeon Desnitzky and Ivan Tretjakoff, ...] // University of Glasgow Archive. GB 248 GUA 30225, 31 Dec 1765. Available at: https://www.gla.ac.uk/collections//details?irn=557137&cat-Type=C&referrer=/results&q=Desnitzky+ (accessed: 16.10.2023).

¹⁰ Storch H. Cours d'économie politique, ou Exposition des principes qui déterminent la prospérité des nations. Paris, 1852.

exchangeable value. As every individual, therefore, endeavours as much as he can, both to employ his capital in the support of domestic industry, and so to direct that industry that its produce maybe of the greatest value; every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain; and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society more effectually than when he really intends to promote it. I have never known much good done by those who affected to trade for the public good. It is an affectation, indeed, not very common among merchants, and very few words need be employed in dissuading them from it^{*11}.

Michael Emmett Brady, a scholar from California, discussed how this above quoted remark from *The Wealth of Nations* showed a change away from Smith's prior thinking about uncertainty. In a private email to the author of this present essay, Michael Brady said that Smith may have read Bernoulli on risk before 1759, yet even so Smith would then have thought that Bernoulli's simple binomial case study had few connections with complex decisions in the world of business.

Smith carried this thought forward and developed it in *The Wealth of Nations*. Smith compared the risk involved in shipping merchandise directly from the Iberian Peninsula to the Baltic Sea as opposed to the risk of shipping the Iberian merchandise first to England and then onward to the Baltic. An English merchant would often choose the latter, Smith said. The English merchant would think it more safe to bring the merchandise first to England although in truth there was no way accurately to compute the differences in the two routes with regard to risk. In preferring to bring this merchandise first to England and thence onward to the Baltic, Smith's imaginary English merchant was led to benefit England, Smith said. The English route would employ English ships which paid English tolls and taxes. Such a merchant would choose an English route by human instinct and by prejudice, not by patriotism, however.

We find correlation for Smith's argument in what now is called the Ellsberg Paradox. Ellsberg thought that people instinctively choose the risk they know as opposed to a risk which they do not know, notwithstanding the possibility that the unknown risk may be the lesser of the two¹².

Michael Brady said that the following passage from Chapter X of Part I of *The Wealth of Nations* also showed how Smith's revised his opinion concerning probability.

"In a perfectly fair lottery, those who draw the prizes ought to gain all that is lost by those who draw the blanks. In a profession, where twenty fail for one that succeeds, that one ought to gain all that should have been gained by the unsuccessful twenty. The counsellor at law, who, perhaps, at near forty years of age, begins to make something by his profession, ought to receive the retribution, not only of his own so tedious and expensive education, but of that of more than twenty others, who are never likely to make any thing by it. How extravagant soever the fees of counsellors at law may sometimes appear, their real retribution is never equal to this.

¹¹ Smith A. Nature and Cause of the Wealth of Nations. Hazleton, 2005. P. 363–364.

¹² *Ellsberg D.* Risk, Ambiguity, and the Savage Axioms // The Quarterly Journal of Economics. 1961. Vol. 75, no. 4. P.643–669.

Compute, in any particular place, what is likely to be annually gained, and what is likely to be annually spent, by all the different workmen in any common trade, such as that of shoemakers or weavers, and you will find that the former sum will generally exceed the latter. But make the same computation with regard to all the counsellors and students of law, in all the different Inns of Court, and you will find that their annual gains bear but a very small proportion to their annual expense, even though you rate the former as high, and the latter as low, as can well be done. The lottery of the law, therefore, is very far from being a perfectly fair lottery; and that as well as many other liberal and honourable professions, is, in point of pecuniary gain, evidently under-recompensed. Those professions keep their level, however, with other occupations; and, notwithstanding these discouragements, all the most generous and liberal spirits are eager to crowd into them. Two different causes contribute to recommend them. First, the desire of the reputation which attends upon superior excellence in any of them; and, secondly, the natural confidence which every man has, more or less, not only in his own abilities, but in his own good fortune. To excel in any profession, in which but few arrive at mediocrity, it is the most decisive mark of what is called genius, or superior talents. The public admiration which attends upon such distinguished abilities makes always a part of their reward; a greater or smaller, in proportion as it is higher or lower in degree. It makes a considerable part of that reward in the profession of physic; a still greater, perhaps, in that of law; in poetry and philosophy it makes almost the whole"13.

Let us agree with Brady that the two passages from *The Wealth of Nations* went beyond the thinking about probability and gambling in the Bernoulli essay on risk¹⁴. If we can agree that the two passages just quoted from *The Wealth of Nations* both went beyond the thinking about probability and gambling in *The Theory of Moral Sentiments*, then we should seek a basis for the new argument in Smith's *The Wealth of Nations*. We might find it in the law.

Speaking of law and counselors at law, as Smith did, here is a recent commentary which might clarify Smith's new thinking about risk and probability. It is a standard book on classical probability by the late Lorraine Daston. If Smith and his Russian students did discuss Bernoulli on risk and probability, maybe they and Smith thought of risk in the same way that they thought about legal questions. Smith taught the students law, after all. We know that Tret'iakov borrowed the library's copy of the proceedings of the Imperial Academy which printed Bernoulli's essay. Tret'iakov did so in connection with his study of physics; nevertheless, he borrowed the book while he was studying law with Smith. Daston said that legal thinking was one early source of argument about probability. Lawyers needed a standard by which to judge the truthfulness of sworn testimony. The lawyers came up with a scale by which witnesses could be judged. In simple terms, they thought social status correlated with truthfulness. A nobleman was more likely to give truthful sworn testimony than a common laborer. They called that being more probable. Mathematicians who took that term up did so by borrowing it from lawyers. It is all not so far from Smith on risk in The Wealth of Nations, come to think of it. Bringing merchandise from Iberia, an English merchant may have thought the familiar route via England was the more probable¹⁵.

¹³ Smith A. Nature and Cause of the Wealth of Nations. Hazleton, 2005. P.92.

¹⁴ Brady M. E. Adam Smith, The Wealth of Nations, and the 'Invisible Hand': A Metaphor for Ambiguity-Uncertainty Aversion by Decision Makers // Scholedge International Journal of Management & Development. 2016. Vol. 3, issue 5. P.97–102.

¹⁵ Daston L. Classical Probability in the Enlightenment. Princeton, 1995.

In sum, Smith probably did read Bernoulli, but if so, then Smith thereafter drifted far from Bernoulli's concept of probability. It was not a case of Smith's disagreeing with Bernoulli. It was a case of Smith's thinking of uncertainties which Bernoulli did not discuss in the essay on risk. In the case of a coin toss, Bernoulli had supposed a simple situation. The coin was tossed, and everyone could agree on the outcome, eagles or tails. In the case of what Smith called the lottery of employment, people could hardly count the varieties of possible outcome nor could people accurately compute their respective chances of success.

Evidence for a link between Smith and Bernoulli is abundant, but it remains merely circumstantial. We have lots of evidence, but there is no secure paper trail.

Let Us Return to Jevons and Moral Expectation

With Jevons, Bernoulli, and moral expectation, we have a secure paper trail. Why did Jevons skip over Daniel Bernoulli's essay on risk? We now know the answer.

In a brilliant article, the Italian scholar Nicola Giocoli set out the story of Jevons' failure to consult the original 1738 article by Daniel Bernoulli. A German scholar named Gossen also preceded Jevons in writing about marginal utility, and Jevons did not read Gossen either. There was, however, a difference between Jevons' not reading Gossen and Jevons' not reading Bernoulli. Jevons apologized about Gossen. Jevons apologized profusely. Jevons said that he knew of Gossen's book, and Jevons said he did not read it because he could not read German. He regretted that omission. He and other British economists had not paid the debt of gratitude due to Gossen. "The day must come when the eyes of those who cannot see will be opened", said Jevons. In the case of not reading Bernoulli, Jevons made no such apology, by contrast. Jevons did not read Gossen, but he knew he did not read him. Not so with Bernoulli¹⁶.

Jevons did not know that he should have read the 1738 essay by Daniel Bernoulli. That was the problem. It was so because Jevons depended instead on two summary versions of the Bernoulli article, one by the famous French mathematician the marquis de Laplace and the other by the English historian Isaac Todhunter. This was what Nicola Giocoli said¹⁷.

Let us entirely accept Giocoli's analysis of Jevons' skipping over Bernoulli's prior work on value and price, but Giocoli did not tell us why Jevons did so. Let us try to guess why, and let us first add an item of our own to Giocoli's excellent analysis. We can be sure that Jevons read Adam Smith very closely. No doubt Jevons thought of Adam Smith when he, Jevons, read Todhunter. Todhunter had this to say in his preface about his own book's subsequent treatment of Daniel Bernoulli.

"The eleventh Chapter relates to Daniel Bernoulli, containing an account of a series of memoirs published chiefly in the volumes of the Academy of Petersburg; the memoirs are remarkable for boldness and originality, the first of them contains the celebrated theory of Moral Expectation"¹⁸.

¹⁶ Jevons W.S. The Theory of Political Economy. London, 1879. P.XXXVIII, XLIII.

¹⁷ Giocoli N. The "True" Hypothesis of Daniel Bernoulli: What Did the Marginalists Really Know? // History of Economic Ideas. 1998. Vol. 6, issue 2. P.7–43.

¹⁸ *Todhunter I.* A History of the Mathematical Theory of Probability from the Time of Pascal to That of Laplace. London, 1865. P.VI.

As you can see, when Todhunter transcribed Bernoulli's Latin discussion, Todhunter used English words which differed from those later used by Jevons. Bernoulli, for instance, distinguished between mathematical expectation, still a common phrase even today, and moral expectation, a phrase reminiscent of another common 19th-century phrase, "moral sciences". Expectation was moral, Todhunter said. There were at least two meanings for the word "moral". Something might be moral because it had to do with right and wrong or something might be moral because it depended on the idiosyncratic circumstances and outlook, in this case, of the particular purchaser or bettor in question. That second one was the meaning of the word in Todhunter's English. A rich man, Bernoulli had said, could spare the cost of his losing a bet more easily than a poor man could, and the rich man experienced the thrill of a winning bet less strongly than a poor man. "Moral" differed for the two men, therefore, not because one man was right and the other one was wrong but because the emotions surrounding the bet and of the winning payoff differed. Those emotions varied according to personal circumstance. The values of the bet and of the payoff depended on their utility to the bettor. That was therefore the meaning of the word "moral". It meant subjective. You could almost say that an invisible hand forced the rich man to bet differently from the poor man.

Bernoulli offered a third and highly mathematical concept of moral expectation¹⁹. Here is a clear summary of the concept of "moral expectation" as it appeared in Daniel Bernoulli's 1738 essay. Joseph Schumpeter made the summary. "The problem is this. A coin is to be tossed n times. X promises Y to pay \$1 if heads turns up on the first throw; \$2 if heads, having failed to turn up the first time, turns up the second time; \$4 if heads, having failed to turn up the first two times, turns up the third time, and so on. The series of Y's possible gains is hence $1, 2, 22, 23, \dots 2n-1$. We derive his mathematical expectation of gain by multiplying each of the possible gains by its probability, that is, if the coin be perfect, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so on. It is seen that this multiplication reduces each item to $\frac{1}{2}$ so that, summing up, we get for Y's total mathematical expectation n/2, and if n is allowed to increase beyond any assigned limit, an expectation greater than any sum we care to mention. Nevertheless, it is the fact that nobody will pay X any considerable sum for it, as the reader can easily find out for himself. Why? Bernoulli thought that all we need to do in order to answer this question is to correct the possible gains by applying his hypothesis to them, which would in fact produce a finite 'moral' expectation in the place of the 'infinite' mathematical one"20.

Daniel Bernoulli also used calculus to compute moral expectation. Here is a summary of that work. "[Daniel Bernoulli used] the differential calculus to represent the diminishing marginal utility of additions to a man's fortune. But it is done upon the basis of an assumption concerning the relationship between this marginal utility and the size of the man's existing fortune which is purely arbitrary (viz., inverse proportionality), and his geometry does no more than illustrate the assumption. Adam Smith would not have flinched from the interpersonal comparison or from the assumption that the marginal utility of an addition to (or, at any rate, subtraction from) the greater fortune was less than that of a similar addition to the smaller fortune^{"21}.

¹⁹ Jordan C. On Daniel Bernoulli's "Moral Expectation" and on a New Conception of Expectation // The American Mathematical Monthly. 1924. Vol. 31, issue 4. P. 183–190.

²⁰ Schumpeter J. History of Economic Analysis. Abington, 1954. P. 288.

²¹ Robertson H. M., Taylor W. L. Adam Smith's Approach to the Theory of Value // The Economic Journal. 1957. Vol. 67, issue 266. P. 181.

In Smith, Jevons found the model for his own argument. Smith wrote plain English prose. If Smith did read Bernoulli (and if Smith did not flinch when reading Bernoulli), Smith then added on his own the notion that "moral sentiments" were those which impelled people to right actions. "Moral expectation" denoted Smith's expectation of right action as opposed to wrong action. Pity was a moral sentiment, Smith said. The rich land-owner was rapacious. Greedy. Lacking in pity. Smith therefore did not expect him to act rightly. No wonder that Smith would use this meaning of "moral" because he was a professor of moral philosophy.

When you read that the rich landowner was greedy, and when you read in the same passage about an invisible hand, you have to understand Smith's background in conventional moral philosophy. Liberty was the absence of restraint, according to the standard argument, and natural liberty was the absence of all restraint except that of nature. A man or woman might wish to jump fifty meters in the air, but they cannot will to do so because nature restrains them. Civil society requires civil and moral restraints in addition to natural restraints. It was often said that we have in nature the ability to kill one another, but civil society depends upon our not doing so. We have to give up to the sovereign our power to arbitrate life and death. The sovereign must establish civil restraints to prevent our killing one another, and the sovereign must do so by force if needed. Moral restraints are those which we impose upon ourselves. They are internal. If you want to smoke, but you do not smoke because your doctor forbade smoking, then you exercise a moral restraint. The unseen hand was the intervention of a natural restraint which rendered redundant the moral restraint, pity, which the rapacious landowner had failed to impose upon his greed.

Jevons may have missed the significance of Bernoulli's mathematical argument about "moral expectation" because Jevons was thinking of Adam Smith. Terminology was a general problem, but the phrase "moral expectation" was especially problematic. Todhunter was Jevons' friend. His friend's choice of the phrase "moral expectation" may have led Jevons to the mistaken view that there was no need to read Bernoulli on risk. Jevons did not see his own innovative notions of value in the summaries of moral expectation and mathematics which he found in Todhunter, and Jevons did not know of this mistake. Maybe also Jevons was thinking of Adam Smith's phrase, "moral sentiments".

In this regard, the word "moral" was like the word "capital". Dr Samuel Johnson defined "capital" as the head of a column. The word was used in that sense by Adam Smith in *The Theory of Moral Sentiments* where he wrote of the Doric capital of a pillar. Eventually and by extension, of course, the initial investment in a firm or enterprise came to be called the firm's capital. Smith, however, never used the word "capital" in that sense in *The Theory of Moral Sentiments*, and he used a different vocabulary in *The Wealth of Nations*. He used words such as "flow" and "stock"²². Much the same confusion happened with terminology in the invention of calculus. Sir Isaac Newton and G. W. Leibniz both invented calculus, and they quarreled over which of them had priority in the invention. Newton won the argument so far as the English-speaking world was concerned, but it was a Pyrrhic victory. Mathematicians preferred the terminology of Leibniz, and we use

²² Endres A. M. A New Institutional Perspective on the Canonical Model: the Case of Capital Markets in The Wealth of Nations // Reflections on the classical canon in economics: essays in honor of Samuel Hollander. London; New York, 2001. P. 57–74.

the German scholar's terms in calculus to this day²³. Jeremy Bentham likewise presented terminological challenges to his readers. There was a real chance that Bentham read Bernoulli, but we must leave that question aside. It is too big a topic for us here. The only point relevant to Adam Smith, Jevons, and Daniel Bernoulli is that Bentham seldom used the word "probability". He used "uncertainty" instead. Of course, he would have read Bernoulli in Latin — if he did read Bernoulli — so Bentham's English words would be his own translations if he were thinking of Bernoulli²⁴.

Jevons, Mathematics, and Gregory King

Reading Adam Smith, Jevons would have found praise for Gregory King who was another possible source of confusion concerning Jevons' concept of marginal utility and another explanation for Jevons's failure to read Daniel Bernoulli. Jevons did not need Bernoulli. Jevons borrowed his supply and price mathematics from Gregory King. Jevons gave a lengthy three-page acknowledgment of this debt, saying that King's "name should be held in honour as one of the fathers of statistical science in England"²⁵. Gregory King was a 17th-century practitioner of political arithmetic who was in the service of Robert Harley who at one time was speaker of the house of commons. Later Harley was ennobled and was one of Queen Anne's principal secretaries of state. Harley kept King's papers secret. As Jevons related at length, the Scottish antiquary George Chalmers discovered King's papers and published them in 1804. Before then, a few bits of King's work were published by Charles Davenant who was himself also in the service of Harley. Jevons recounted all this, citing the various authorities²⁶. The 1804 book by Chalmers is well worth reading²⁷.

There was a tradition of early borrowing from King and Davenant as was clearly established in 1987 by Anthony M. Endres. There was a clear connection between Davenant and King, on the one hand, and Adam Smith, on the other. Smith read Davenant carefully and quoted Gregory King from Davenant's quotation of King. Jevons, by the way, read King in the complete edition published by George Chalmers, but Jevons also read and admired Davenant and spoke highly of his books. So did Alfred Marshall²⁸. I am grateful to Professor Endres for corresponding with me on this point.

Latin was not an obstacle to Jevons, but, again, maybe mathematics was an obstacle. Jevons may not have read Bernoulli in the original because Jevons was not skillful enough in higher mathematics. The distinguished University of Chicago economist Milton Friedman said this of Jevons. "Despite his deserved reputation as a pioneer in economic statistics, Jevons was almost consistently wrong in his empirical predictions"²⁹.

²³ Bardi J. S. The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time. New York, 2007.

²⁴ Bentham J. An Introduction to the Principles of Morals and Legislation (1781). Kitchener, 2000.

²⁵ Jevons W. S. Political Economy. New York, Appleton, 1880. P. 154.

²⁶ Ibid. P. 155.

²⁷ *Chalmers G., King G.* An estimate of the comparative strength of Britain during the present and four preceding reigns; <...>. Hale, 1804.

²⁸ Endres A. The King-Davenant "Law" in Classical Economics' // History of Political Economy. 1987. Vol. 19, issue 4. P. 621–638.

²⁹ *Friedman M.* Bimetallism Revisited // The Journal of Economic Perspectives. 1990. Vol. 4, issue 4. P.98.

In passing up Daniel Bernoulli's famous essay on the Saint Petersburg Paradox, Jevons also passed up the development of classical probability. The Bernoulli family made crucial initial contributions to that development. Many historians of mathematics also attribute to Bernoulli family important refinements in the calculus, and the Bernoulli family corresponded with Leibniz on that point³⁰. We can understand why Jevons was so consistently wrong in his empirical predictions. Let us reconstruct the way he did his mathematics. In sum, Jevons stayed with the mathematical methods which he inherited from his 17th-century British predecessors. Take Jevons' book on coal, for instance, or his book on the price of gold. He did simple arithmetic and some algebra in those books, of course, and he displayed his data in his tables. The coal book therefore looked on every page nothing like Bernoulli's essay on the Saint Petersburg Paradox. Jevons' books on coal or gold instead resembled very much the work of Gregory King. Jevons, for instance, chose his time periods by using educated guess work and his skilled eye — in other words, by 17th-century methods.

As for arithmetic, Jevons must have used an abacus to compile his data. He probably also used a slide rule, a ruler with a sliding central strip marked with logarithmic scales. There is no evidence that Gregory King used a slide rule, although he might have done so, and other 17th-century people did use that invention. Jevons' arithmetic was not the problem, however. Like the ancient Greeks when they practiced higher mathematics, Jevons sought for harmonies and ratios which were implicit in nature and the study of which would yield transcendent moral truths. He sought these transcendental moral truths, for instance, in his study of coal. Jevons' transcendent morality was British patriotism. Coal was important because it was a means of national greatness. The sure basis of national greatness, however, was the superior merit of British people. Similarly, Jevons thought that the fall in the value of gold would increase the number of British colonies and spread the English language³¹.

Jevons ignored Daniel Bernoulli, but Jevons also ignored many other previous writers in whose works he might have found inspiration for the concept of marginal utility. The number of such previous writers was legion. In a brilliant article, Emil Kauder demonstrated that Aristotle and the medieval schoolmen had early notions of marginal utility. Many later European writers were similar possible sources of inspiration. Kauder blamed Adam Smith for not taking up the notion of marginal utility which was clear in those previous writers. Kauder wrote as follows. It was a tragedy that these previous writers wrote in vain, he said. It was a tragedy that they were forgotten. When Adam Smith failed to name them and to champion their ideas, Smith made, said Kauder, "waste and rubbish out of the thinking of 2,000 years. The chance to start in 1776 instead of 1870 with a more correct knowledge of value principles had been missed"³².

If we conclude that Smith read Bernoulli, we therefore face another paradox. We must then also grant that what Kauder said of Smith was true. Smith in 1759 did not take up the challenge of Bernoulli's mathematical probability. Smith's notion of an "invisible hand" substituted for Bernoulli's notion of "moral expectation". Smith offered a notion of "moral

³⁰ Stigler S. A. The History of Statistics. Cambridge, 1986. P. 64.

³¹ *Jevons W.S.* The Coal Question: An Inquiry Concerning the Progress of the Nation, and the Probable Exhaustion of Our Coal-Mines, London, 1866.

³² Kauder E. Genesis of the Marginal Utility Theory: From Aristotle to the End of the Eighteenth Century' // The Economic Journal. 1953. Vol. 63, issue 251. P.638–650.

expectation" which came from his own discipline, moral philosophy. Kauder took up *The Wealth of Nations*, 1776, and there, as Kauder noted, Smith's views roamed even further from Bernoulli. We noted earlier a similar point from Michael Brady. Smith discussed a utility in use versus a utility in exchange. Smith gave the famous examples of water and diamonds. The former had great utility in use but little utility in exchange. The latter had little utility in use but great utility in exchange³³.

Jevons ignored most of the previous writers in whose works he could have found the seed of marginal utility, but he did not ignore them all. As we have seen, Jevons picked out Gregory King as the best of them. King used only simple arithmetic in his analysis of supply and price, mind. King said that the price of bread rose merely on the fear of shortage, not on any actual shortage. In other words, a change of sentiment in the minds of buyers was enough to alter value³⁴.

By choosing Gregory King over Daniel Bernoulli, Jevons made his paradoxical choice. In the same way that Jevons chose Gregory King over Bernoulli, Marshall chose Jevons over Bernoulli. That was also paradoxical. It was paradoxical because Marshall and Jevons missed the more mathematical version of marginal utility which Bernoulli had published in 1738, yet Jevons and Marshall were said to have added not only marginal utility but also mathematics to economics³⁵. Jevons and Marshall made a paradoxical choice, but it was decisive. It was also an English choice, a British choice. Marshall set the canon of received works read by English-speaking economists in the later 19th-century. The canon narrowed and altered the way that later English-speaking economists read previous economic texts. English-speaking economists saw *The Wealth of Nations* as their path-breaking founding document of their new British school of economics, not as the capstone of a rich, prior, and pan-European tradition of writings in political economy. English-speaking economists often simply ignored the wide range of sophisticated and skillful European 17th-century and early 18th-century writers. This was all paradoxical³⁶.

A Final Saint Petersburg Paradox

Jevons became famous for his merger of mathematics and economics, but that fame was misplaced. Jevons avoided advanced modern higher mathematics. This effect lasted long because of Alfred Marshall who was an accomplished mathematician and who studied in Germany in his youth. Marshall, however, came to share Jevons' opinion about the proper role of mathematics in economics, and Marshall made the opinion so crystal clear that it long influenced English-speaking economists. It was a final Saint Peterburg Paradox. Marshall wrote as follows. "The chief use of pure mathematics in economic questions seems to be in helping a person to write down quickly, shortly and exactly, some of his thoughts for his own use: and to make sure that he has enough, and only enough, premises

³³ *Gupta A. K. D.* Adam Smith on Value // Indian Economic Review. 1960. Vol. 5, issue 2. P. 105–115.

³⁴ *Taylor J.A.*: 1) British Empiricism and Early Political Economy: Gregory King's Estimates of National Wealth and Population. Westport, 2005; 2) Was Seventeenth-century British Political Arithmetic a Precursor of Nineteenth-century Economic Science? // Terra Economicus. 2021. Vol. 21, issue 1. P. 32–46.

³⁵ *Schabas M. A.* World Ruled by Number: William Stanley Jevons and the Rise of Mathematical Economics. Princeton, 1990.

³⁶ *Taylor J.A.*: 1) Adam Smith's seventeenth-century sources // Terra Economicus. 2019. Vol. 17, issue 3. P. 78–88; 2) Adam Smit i neoliberal'naia ekonomika. St. Petersburg, 2006. (In Russian)

for his conclusions (i. e., that his equations are neither more nor less in number than his unknowns). But when a great many symbols have to be used, they become very laborious to anyone but the writer himself^{"37}.

Conclusion

We are at the end of this essay. Maybe Daniel Bernoulli's seminal 1738 essay on risk influenced Adam Smith's Smith's notion of an invisible hand. This was possible, even likely, and it was important because the notions of Adam Smith echoed through so much subsequent economic literature, and they especially influenced the work on marginal utility by William Stanley Jevons and Alfred Marshall. The evidence about Bernoulli's influence on Smith is circumstantial so the matter is not beyond reasonable doubt. Nevertheless, as Sherlock Holmes said, quoting Henry David Thoreau, circumstantial evidence can occasionally be very convincing as when you find a trout in the milk.

References

- Alekseev M. P. Adam Smith and his Russian admirers of the eighteenth century. *Econ Journal Watch Scholarly Comments on Academic Economics*, 2018, vol. 15, issue 3, pp. 354–364.
- Bardi J.S. *The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time.* New York, Basic Books, 2007, 304 p.
- Brady M. E. Adam Smith, The Wealth of Nations, and the 'Invisible Hand': A Metaphor for Ambiguity-Uncertainty Aversion by Decision Makers. *Scholedge International Journal of Management & Development*, 2016, vol. 3, issue 5, pp. 97–102.
- Daston L. Classical Probability in the Enlightenment. Princeton, Princeton University Press, 1995, 448 p.
- Ellsberg D. Risk, Ambiguity, and the Savage Axioms. *The Quarterly Journal of Economics*, 1961, vol. 75, no. 4, pp. 643–669.
- Endres A. M. A New Institutional Perspective on the Canonical Model: the Case of Capital Markets in The Wealth of Nations. *Reflections on the classical canon in economics: essays in honor of Samuel Hollander*. London; New York, Routledge, 2001, pp. 57–74.
- Endres A. The King-Davenant "Law" in Classical Economics. *History of Political Economy*, 1987, vol. 19, issue 4, pp. 621–638.
- Friedman M. Bimetallism Revisited. The Journal of Economic Perspectives, 1990, vol. 4, issue 4, pp. 85-104.
- Giocoli N. The "True" Hypothesis of Daniel Bernoulli: What Did the Marginalists Really Know? *History of Economic Ideas*, 1998, vol. 6, issue 2, pp.7–43.
- Gupta A.K. D. Adam Smith on Value. Indian Economic Review, 1960, vol. 5, issue 2, pp. 105–115.
- Jordan C. On Daniel Bernoulli's "Moral Expectation" and on a New Conception of Expectatio'. *The American Mathematical Monthly*, 1924, vol. 31, issue 4, pp. 183–190.
- Kauder E. Genesis of the Marginal Utility Theory: From Aristotle to the End of the Eighteenth Century. *The Economic Journal*, 1953, vol. 63, issue 251, pp. 638–650.
- Kudryavtsev A. A. St. Petersburg Paradox and its Significance for Economic Theory. Vestnik of Saint Petersburg State University. Seriia 5. Economics, 2013, issue 3, pp.41–55.
- Peart S. J., Levy D. M. Menger and Jevons: beliefs and things. *The Review of Austrian Economics*, 2023, vol. 36, issue 2, pp. 271–287.
- Robertson H. M., Taylor W. L. Adam Smith's Approach to the Theory of Value. *The Economic Journal*, 1957, vol. 67, no. 266, pp. 181–198.
- Savage L. J. The Foundations of Statistics. New York, Dover, 1972, 352 p.
- Schabas M.A. World Ruled by Number: William Stanley Jevons and the Rise of Mathematical Economics. Princeton, Princeton University Press, 1990, 204 p.

³⁷ Marshall A. Principles of Economics. P.7.

Schumpeter J. History of Economic Analysis. Abington, Routledge, 1954, 1260 p.

Stigler S. A. The History of Statistics. Cambridge, Harvard University Press, 1986, 410 p.

Taylor J. A. Adam Smit i neoliberal'naia ekonomika. St. Petersburg, St. Petersburg University Press, 2006, 100 p. (In Russian)

Taylor J. A. Adam Smith's seventeenth-century sources. Terra Economicus, 2019, vol. 17, issue 3, pp. 78-88.

- Taylor J. A. British Empiricism and Early Political Economy: Gregory King's Estimates of National Wealth and Population. Westport, Praeger, 2005, 193 p.
- Taylor J. A. Was Seventeenth-century British Political Arithmetic a Precursor of Nineteenth-century Economic Science? *Terra Economicus*, 2021, vol. 21, issue 1, pp. 32–46.
- Todhunter I. A History of the Mathematical Theory of Probability from the Time of Pascal to That of Laplace. London, Macmillan, 1865, 624 p.

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